

SIMPLIFYING RADICALS

#29

Sometimes it is convenient to leave square roots in radical form instead of using a calculator to find approximations (decimal values). Look for perfect squares (i.e., 4, 9, 16, 25, 36, 49, ...) as **factors** of the number that is inside the radical sign (**radicand**) and take the square root of any perfect square factor. Multiply the root of the perfect square times the reduced radical. When there is an existing value that multiplies the radical, multiply any root(s) times that value.

For example:

$$\sqrt{9} = 3$$

$$5\sqrt{9} = 5 \cdot 3 = 15$$

$$\sqrt{18} = \sqrt{9 \cdot 2} = \sqrt{9} \cdot \sqrt{2} = 3\sqrt{2}$$

$$3\sqrt{98} = 3\sqrt{49 \cdot 2} = 3 \cdot 7\sqrt{2} = 21\sqrt{2}$$

$$\sqrt{80} = \sqrt{16 \cdot 5} = \sqrt{16} \cdot \sqrt{5} = 4\sqrt{5}$$

$$\sqrt{45} + 4\sqrt{20} = \sqrt{9 \cdot 5} + 4\sqrt{4 \cdot 5} = 3\sqrt{5} + 4 \cdot 2\sqrt{5} = 11\sqrt{5}$$

When there are no more perfect square factors inside the radical sign, the product of the whole number (or fraction) and the remaining radical is said to be in **simple radical form**.

Simple radical form does not allow radicals in the denominator of a fraction. If there is a radical in the denominator, **rationalize the denominator** by multiplying the numerator and denominator of the fraction by the radical in the original denominator. Then simplify the remaining fraction. Examples:

$$\frac{2}{\sqrt{2}} = \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$$

$$\frac{4\sqrt{5}}{\sqrt{6}} = \frac{4\sqrt{5}}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{4\sqrt{30}}{6} = \frac{2\sqrt{30}}{3}$$

In the first example, $\sqrt{2} \cdot \sqrt{2} = \sqrt{4} = 2$ and $\frac{2}{2} = 1$. In the second example,

$$\sqrt{6} \cdot \sqrt{6} = \sqrt{36} = 6 \text{ and } \frac{4}{6} = \frac{2}{3}.$$

The rules for radicals used in the above examples are shown below. Assume that the variables represent non-negative numbers.

$$(1) \sqrt{x} \cdot \sqrt{y} = \sqrt{xy}$$

$$(2) \sqrt{x \cdot y} = \sqrt{x} \cdot \sqrt{y}$$

$$(3) \frac{\sqrt{x}}{\sqrt{y}} = \sqrt{\frac{x}{y}}$$

$$(4) \sqrt{x^2} = (\sqrt{x})^2 = x$$

$$(5) a\sqrt{x} + b\sqrt{x} = (a + b)\sqrt{x}$$

Write each expression in simple radical (square root) form.

- | | | | |
|------------------------------------|------------------------------------|----------------------------------|----------------------------------|
| 1. $\sqrt{32}$ | 2. $\sqrt{28}$ | 3. $\sqrt{54}$ | 4. $\sqrt{68}$ |
| 5. $2\sqrt{24}$ | 6. $5\sqrt{90}$ | 7. $6\sqrt{132}$ | 8. $5\sqrt{200}$ |
| 9. $2\sqrt{6} \cdot 3\sqrt{2}$ | 10. $3\sqrt{12} \cdot 2\sqrt{3}$ | 11. $\frac{\sqrt{12}}{\sqrt{3}}$ | 12. $\frac{\sqrt{20}}{\sqrt{5}}$ |
| 13. $\frac{8\sqrt{12}}{2\sqrt{3}}$ | 14. $\frac{14\sqrt{8}}{7\sqrt{2}}$ | 15. $\frac{2}{\sqrt{3}}$ | 16. $\frac{4}{\sqrt{5}}$ |
| 17. $\frac{6}{\sqrt{3}}$ | 18. $\frac{2\sqrt{3}}{\sqrt{6}}$ | 19. $2\sqrt{3} + 3\sqrt{12}$ | 20. $4\sqrt{12} - 2\sqrt{3}$ |
| 21. $6\sqrt{3} + 2\sqrt{27}$ | 22. $2\sqrt{45} - 2\sqrt{5}$ | 23. $2\sqrt{8} - \sqrt{18}$ | 24. $3\sqrt{48} - 4\sqrt{27}$ |

Answers

- | | | | | |
|---------------------------|------------------|-----------------|-----------------|---------------------------|
| 1. $4\sqrt{2}$ | 2. $2\sqrt{7}$ | 3. $3\sqrt{6}$ | 4. $2\sqrt{17}$ | 5. $4\sqrt{6}$ |
| 6. $15\sqrt{10}$ | 7. $12\sqrt{33}$ | 8. $50\sqrt{2}$ | 9. $12\sqrt{3}$ | 10. 36 |
| 11. 2 | 12. 2 | 13. 8 | 14. 4 | 15. $\frac{2\sqrt{3}}{3}$ |
| 16. $\frac{4\sqrt{5}}{5}$ | 17. $2\sqrt{3}$ | 18. $\sqrt{2}$ | 19. $8\sqrt{3}$ | 20. $6\sqrt{3}$ |
| 21. $12\sqrt{3}$ | 22. $4\sqrt{5}$ | 23. $\sqrt{2}$ | 24. 0 | |